RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4763:** Mechanics 3

Mark Scheme for June 2011

Oxford Cambridge and RSA Examinations

1 (i)	$\frac{dx}{dt} = A\omega\cos\omega t$ $\frac{d^2x}{dt^2} = -A\omega^2\sin\omega t$ $= -\omega^2(A\sin\omega t) = -\omega^2 x$ $\left(\frac{dx}{dt}\right)^2 = A^2\omega^2\cos^2\omega t = A^2\omega^2(1-\sin^2\omega t)$ $= \omega^2(A^2 - A^2\sin^2\omega t) = \omega^2(A^2 - x^2)$	B1 M1 E1 M1 E1 **5**	 Obtaining second derivative Using $\cos^2\omega t = 1-\sin^2\omega t$
(ii)	$1.2^2 = \omega^2(A^2 - 0.7^2)$ $0.75^2 = \omega^2(A^2 - 2^2)$ $\frac{A^2 - 0.49}{A^2 - 4} = \frac{1.2^2}{0.75^2}$ $A^2 - 0.49 = 2.56(A^2 - 4)$ $9.75 = 1.56A^2$ $A^2 = 6.25$ Amplitude is 2.5 m $1.44 = \omega^2(2.5^2 - 0.7^2)$ $\omega = 0.5$ Period is $\frac{2\pi}{\omega} = \frac{2\pi}{0.5}$ $= 4\pi = 12.6$ s (3 sf)	M1 A1 M1 E1 M1 A1 **6**	Using $v^2 = \omega^2(A^2 - x^2)$ Two correct equations (M0 if $x = 7.3$ used, etc) Eliminating ω or Eliminating A or Substituting $A = 2.5$ into both equations Correctly shown Using $\frac{2\pi}{\omega}$
(iii)	Maximum speed is $A\omega = 1.25 \text{ m s}^{-1}$	B1 **1**	ft only if greater than 1.2
(iv)	Magnitude is $0.5^2 \times 1.6$ $= 0.4 \text{ m s}^{-2}$ Direction is upwards	M1 A1 B1 **3**	 Accept -0.4 B0 for just 'towards centre'
(v)	$x = 2.5\sin(0.5t)$ When $x = -2$, $t = -1.855$ (or 10.71) When $x = 1$, $t = 0.823$ (or 13.39) Time taken is $0.823 - (-1.855)$ $= 2.68$ s (3 sf)	B1 M1 A1 **3**	or $x = 2.5\cos(0.5t)$ or $t = (\pm)\ 4.996$ or $t = (\pm)\ 2.319$ Correct strategy for finding time (must use radians) (ft is $1.3388/\omega$)

2(a)(i)	$0.6+0.2\times 9.8=0.2\times\frac{u^2}{3.2}$ Speed is $6.4\ \mathrm{m\,s^{-1}}$	M1 A1 A1 **3**	For acceleration $\frac{u^2}{3.2}$
(ii)	(*A*) $\frac{1}{2}m(v^2-u^2)=m\times 9.8(3.2+3.2\cos 60°)$ $v^2=135.04$ Radial component is $\frac{v^2}{3.2}=42.2\ \mathrm{m\,s^{-2}}$ Tangential component is $g\sin 60°$ $=8.49\ \mathrm{m\,s^{-2}}$ (3 sf)	M1 A1 A1 M1 A1 **5**	Equation involving KE and PE (ft is $29.4+\frac{u^2}{3.2}$) M1A0 for $g\cos 60°$ M0 for $mg\sin 60°$ *If radial and tangential components are interchanged, withhold first A1*
	(*B*) $T-mg\cos 60°=ma$ $T-0.2\times 9.8\cos 60°=0.2\times 42.2$ Tension is 9.42 N	M1 A1 A1 cao **3**	Radial equation (three terms) (Allow M1 for $T-mg=ma$) *This M1 can be awarded in (A)* *ft dependent on M1 for energy in (A)* *SC* If $60°$ with upward vertical, *(A)* M1A0A0 M1A1 (8.49) *(B)* M1A1A1 (3.54)
(b)(i)	$T\cos 36°+0.75\sin 36°=0.2\times 9.8$ Tension is 1.88 N (3 sf)	M1 A1 **2**	Resolving vertically (three terms) *Allow sin/cos confusion, but both T and R must be resolved*
(ii)	Angular speed $\omega=\frac{2\pi}{1.8}$ ($=3.491$) $T\sin 36°-0.75\cos 36°=0.2r\left(\frac{2\pi}{1.8}\right)^2$ $r=0.204$ Length of string is $\frac{r}{\sin 36°}$ $=0.347$ m (3 sf)	B1 M1 A1 M1 A1 cao **5**	Or $v=\frac{2\pi r}{1.8}$ Horiz eqn involving $r\omega^2$ or v^2/r Equation for r (or l) *Dependent on previous M1*

3 (i)	Elastic energy is $\frac{1}{2}\times\frac{573.3}{3.9}\times 0.9^2$ $= 59.535$ J	M1 A1 **2**	Allow one error *(Allow 60 A0 for 59)*
(ii)	Length of string at bottom is $2\sqrt{1.8^2+2.4^2}$ $(=6)$ $\frac{1}{2}\times\frac{573.3}{3.9}\times(2.1^2-0.9^2) = m\times 9.8\times 1.8$ $324.135 - 59.535 = 17.64m$ Mass is 15 kg	M1 M1 B1B1 E1 **5**	Finding length of string (or half-string) Equation involving EE and PE For change in EE and change in PE
(iii)	Length of string is $2\sqrt{1.0^2+2.4^2} = 5.2$ Tension $T = \frac{573.3}{3.9}\times 1.3$ $(=191.1)$ $2T\sin\alpha - mg = 2\times 191.1\times\frac{1.0}{2.6} - 15\times 9.8$ $= 147 - 147$ $= 0$, hence it is in equilibrium	M1 A1 M1 E1 **4**	Finding tension (via Hooke's law) Finding vertical component of tension Give A1 for $T = 191.1$ obtained from resolving vertically *SC* If 573.3 is used as stiffness: (i) M1A0 (ii) M1M1B0B1E0 (iii) M1A1 (745.29) M1E0
(iv)	$[\,8\pi^2h^3\,] = \mathrm{L}^3$, $[\,8h^3 - ad^2\,] = \mathrm{L}^3$ So $\frac{8\pi^2h^3}{8h^3-ad^2}$ is dimensionless	E1 **1**	*Condone* '$\mathrm{L}^3/\mathrm{L}^3 = 0$, *dimensionless'* *But E0 for* $\frac{\mathrm{L}^3}{\mathrm{L}^3-\mathrm{L}^3} = \frac{\mathrm{L}^3}{0}$
(v)	$\mathrm{T} = \mathrm{M}^{\alpha}\,\mathrm{L}^{\beta}\,(\mathrm{M\,L\,T}^{-2})^{\gamma}$ $\gamma = -\frac{1}{2}$ $\alpha+\gamma = 0$, so $\alpha = \frac{1}{2}$ $\beta+\gamma = 0$, so $\beta = \frac{1}{2}$	B1 B1 B1 B1 **4**	For $[\,\lambda\,] = \mathrm{M\,L\,T}^{-2}$ If γ is wrong but non-zero, give B1 ft for $\alpha = \beta = -\gamma$
(vi)	$a = 3.9$, $\lambda = 573.3$, $d = 4.8$, $h = 2.6$, $m = 15$ Period is $\sqrt{\frac{8\pi^2h^3}{8h^3-ad^2}}\; m^{\frac{1}{2}}\,a^{\frac{1}{2}}\,\lambda^{-\frac{1}{2}} = 1.67$ s (3 sf)	M1 A1 cao **2**	Using formula with numerical α, β, γ (must use the complete formula)

4 (i)	Area is $\int_0^3 (x^2+5)\,dx$	M1	For $\int (x^2+5)\,dx$
	$= \left[\frac{1}{3}x^3 + 5x \right]_0^3 \quad (=24)$	A1	For $\frac{1}{3}x^3+5x$
	$\int xy\,dx = \int_0^3 (x^3+5x)\,dx$	M1	For $\int xy\,dx$
	$= \left[\frac{1}{4}x^4 + \frac{5}{2}x^2 \right]_0^3 \quad (=\frac{171}{4})$	A1	For $\frac{1}{4}x^4+\frac{5}{2}x^2$
	$\bar{x} = \frac{42.75}{24} = \frac{57}{32} = 1.78125$	A1	
	$\int \frac{1}{2}y^2\,dx = \int_0^3 \frac{1}{2}(x^4+10x^2+25)\,dx$	M1	For $\int y^2\,dx$
	$= \left[\frac{1}{10}x^5 + \frac{5}{3}x^3 + \frac{25}{2}x \right]_0^3 \quad (=106.8)$	A2	For $\frac{1}{10}x^5+\frac{5}{3}x^3+\frac{25}{2}x$ Give A1 for two terms correct
	$\bar{y} = \frac{106.8}{24} = \frac{89}{20} = 4.45$	A1 **9**	
(ii)	Volume is $\int \pi x^2\,dy = \int_5^{14} \pi(y-5)\,dy$	M1	For $\int (y-5)\,dy$
	$= \pi\left[\frac{1}{2}y^2 - 5y \right]_5^{14} \quad (=40.5\pi)$	A1	For $\left[\frac{1}{2}y^2-5y \right]_5^{14}$
	$\int \pi x^2 y\,dy = \int_5^{14} \pi(y^2-5y)\,dy$	M1	For $\int x^2 y\,dx$
	$= \pi\left[\frac{1}{3}y^3 - \frac{5}{2}y^2 \right]_5^{14} \quad (=445.5\pi)$	A1	For $\frac{1}{3}y^3-\frac{5}{2}y^2$
	$\bar{y} = \frac{445.5\pi}{40.5\pi}$	M1	*Dependent on previous M1M1*
	$=11$	A1 **6**	
(iii)	Volume of whole cylinder is $\pi\times 3^2\times 14 = 126\pi$		
	$126\pi\times 7 = 40.5\pi\times 11 + (126\pi-40.5\pi)\times \bar{y}_A$	M1 A1	Using formula for composite body
	$\bar{y}_A = \frac{126\pi\times 7 - 40.5\pi\times 11}{126\pi - 40.5\pi}$		
	$= \frac{97}{19} = 5.105 \quad$ (4 sf)	A1 cao **3**	